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EULER'S THEOREM.

The number of faces and summits in any polyhedron taken together exceeds by two the number of its edges.

Proof.

First Case. If all the faces are triangles; then by Descartes's theorem,

$$F = 2(S - 2).$$

But also $2E = 3F$, since each edge belongs to two faces, and so we get a triangle for every time 3 is contained in $2E$. By adding, we have

$$2E = 2F + 2(S - 2),$$

that is,

$$F + S = E + 2.$$

Second Case. If not, all the faces are triangles. Since to any summit go as many faces as edges, we may replace any polygonal face by a pyramidal summit without changing the equality or inequality relation of $F + S$ to $E + 2$; for such replacement only adds the same number to F as to E and changes one face to a summit. But after all polygonal faces have been so replaced, $F + S = E + 2$ by our first case. Therefore the relation is always equality.

[These theorems are substantially one; the second is also due to Descartes, having been published in 1860 in his *Œuvres Inédites*.—*W. M. T.*]

 PROOF OF A PROPOSITION IN MODERN GEOMETRY.

By MR. R. D. BOHANNAN, University of Virginia.

A, B are any two points on the curve which is the locus of the intersection of corresponding rays of two homographic ray-systems. If A, B be made the centres of two ray-systems whose corresponding rays intersect on this curve, these two ray-systems are homographic.

The curve which is the locus of corresponding rays of two homographic ray-systems is of the second degree and passes through the two ray-centres. Being of the second degree, it is fixed by fixing on it five points A, B, C, D, E . Take on it any sixth point F . The three rays BC, BD, BE may be taken arbitrarily to correspond to the three rays AC, AD, AE . Suppose the ray BK corresponding to BF does not intersect AF in F , but in K . Then we have two curves of the second degree, one passing through the six points A, B, C, D, E, F and the other passing through the six points A, B, C, D, E, K . But the curves have five points in common. Thus K, F are coincident. Therefore, etc.

If $AC, AD, AE, AF, BC, BD, BE, BF$ are homographic, the six points A, B, C, D, E, F , no three of them being on a straight line, lie on a curve of the second degree. Thus, by the preceding proposition, the rays drawn from any two of the six points A, B, C, D, E, F to the other four are homographic systems.

Chasles proves these propositions in the inverse order to that here given. See *Traité de Géométrie Supérieure*, Chapter XXV, Arts. 544, 416.



SOLUTIONS OF EXERCISES.

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It is assumed that when a gate in a water-pipe is closing the pressure increases uniformly and the discharge decreases uniformly. Investigate an expression for the shortest safe time for closing the gate on the basis of these hypotheses: given the length of the pipe, the velocity of the stream, the working pressure, and the greatest admissible pressure to which the pipe may be exposed.

[*W. M. Thornton.*]

SOLUTION.

Put L for the length of the pipe,

V velocity of the stream,

Ω sectional area of the pipe,

g acceleration due to gravity,

D heaviness of water,

p working pressure in the pipe,

P highest admissible pressure in the pipe,

T required time in seconds.

The kinetic energy of the stream is

$$\frac{D}{2g} \cdot L\Omega V^2.$$

The velocity and pressure at t seconds after the closing of the valve begins are

$$V_t = V \left[1 - \frac{t}{T} \right], \quad p_t = p + (P - p) \frac{t}{T};$$

and the work done by the stream in entering this region of high pressure is